



FROM GENOMES TO SUPERGENOMES

HOW TO DEAL WITH BETWEENNESS

FABIAN EXTERNBRINK

SCADS & BIOINFORMATIK, LEIPZIG

FABIAN@BIOINF.UNI-LEIPZIG.DE

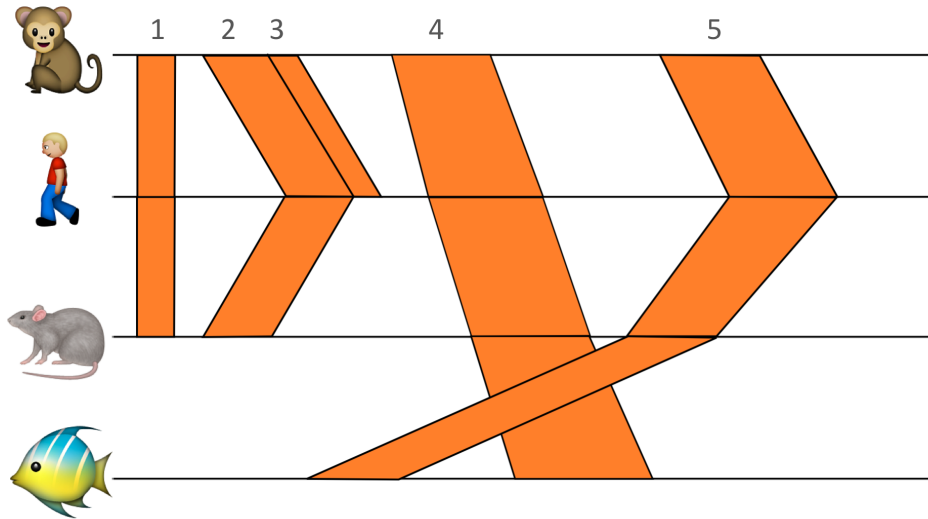
www.scads.de

GenomeRing: alignment visualization based on SuperGenome coordinates

A. Herbig[†], G. Jäger[†], F. Battke[†] and K. Nieselt*

Center for Bioinformatics Tübingen, Faculty of Science, University of Tübingen, Sand 14, 72076 Tübingen, Germany

A Supergenome is a common coordinate system for all genomes in a multiple alignment.



- Multiple alignments
- Alignment blocks i.e. local best alignments
- Evolutionary events change the order
- Task:
 - Order the Blocks to create a common coordinate system

TOTAL ORDERING PROBLEM*

J. OPATRNY†

Abstract. The problem of finding a total ordering of a finite set satisfying a given set of in-between restrictions is considered. It is shown that the problem is *NP*-complete.

Key words. algorithms, computational complexity, total ordering, *NP*-completeness

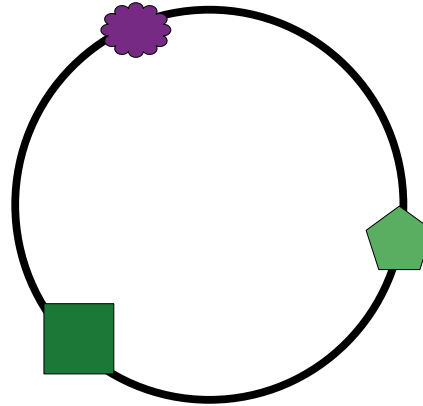
Deciding Problem

Given a finite set X and a collection $C \subseteq X^3$, is there a total order on X such that $(i, j, k) \in C$ either $i < j < k$ or $i > j > k$?

Optimization Problem

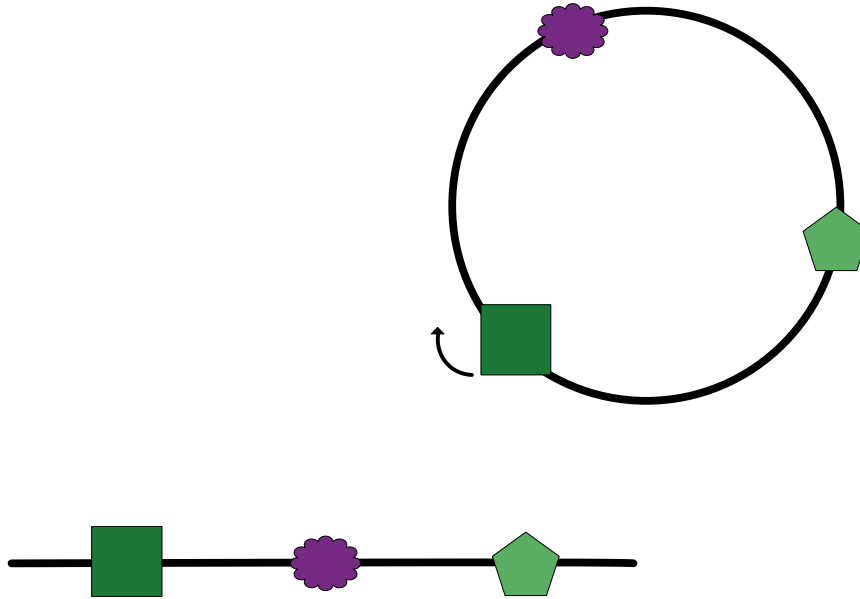
Given a finite set X and a collection $C \subseteq X^3$, find a maximal subset S from C , for which the decision problem w.r.t. S is true.

TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS

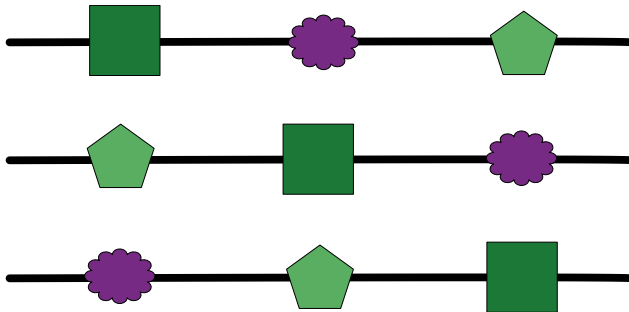
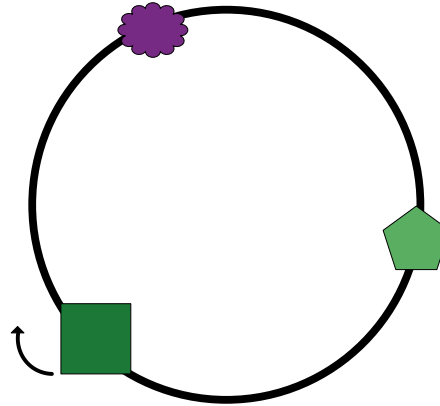


- Given a circular RNA molecule
- Different marker may exist on the RNA molecule
- Question:
 - What was the linear transcript?
 - Linear order of the marker?

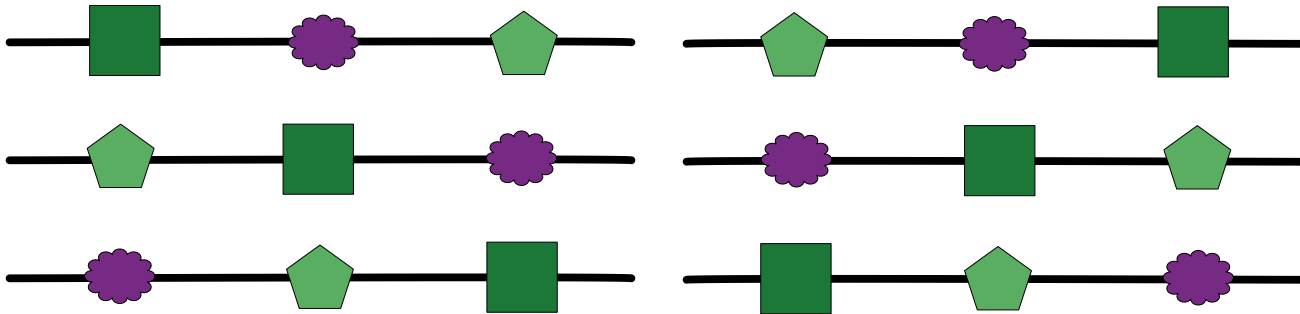
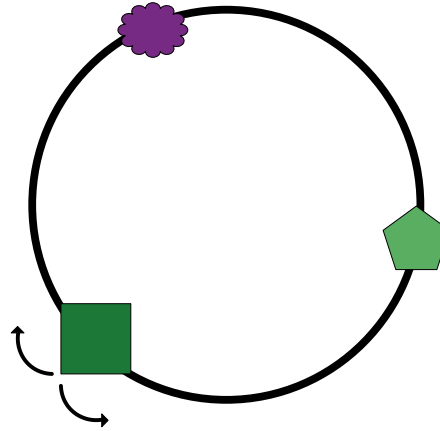
TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS



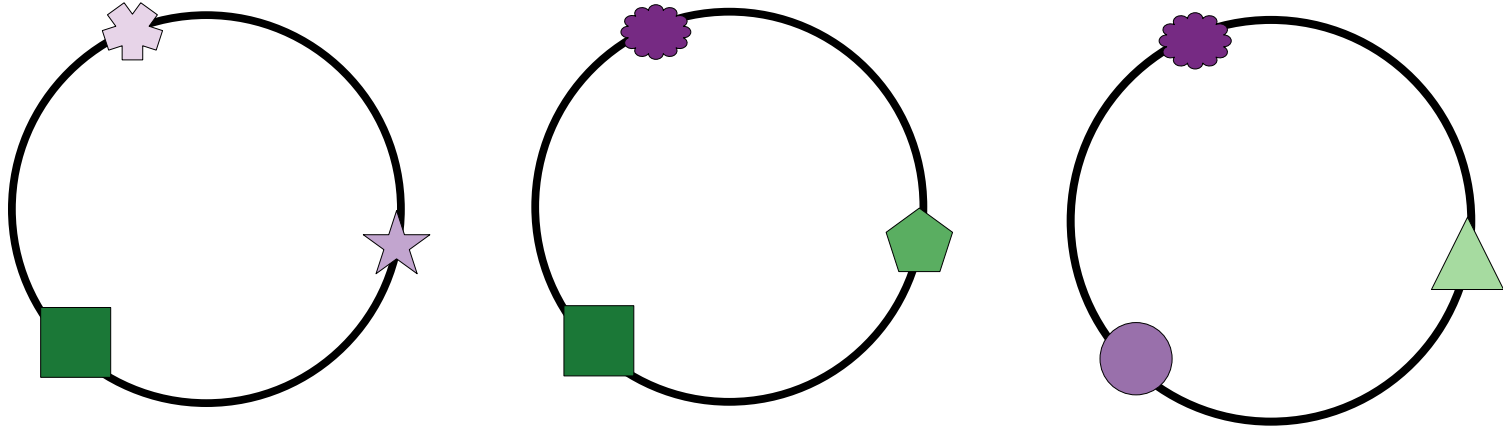
TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS



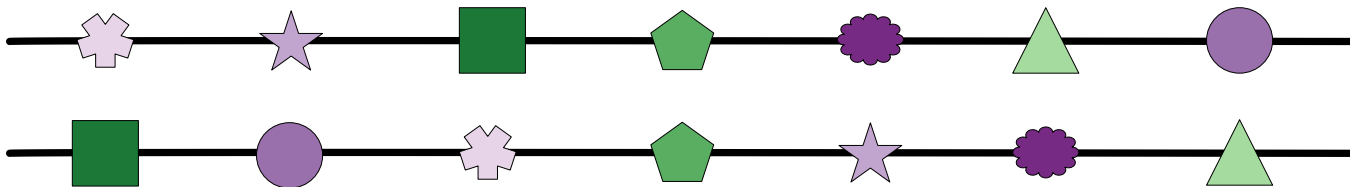
TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS



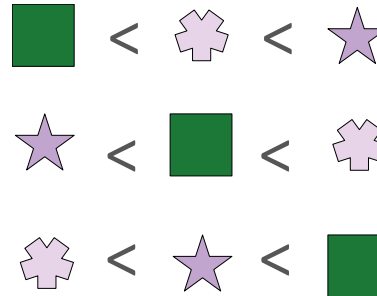
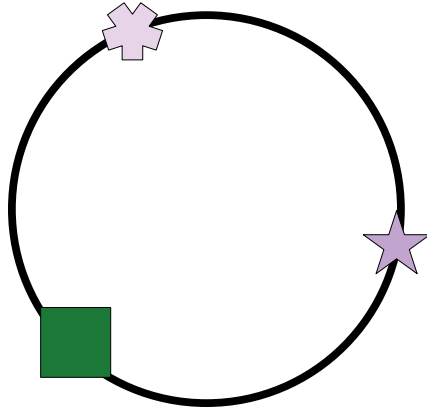
TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS



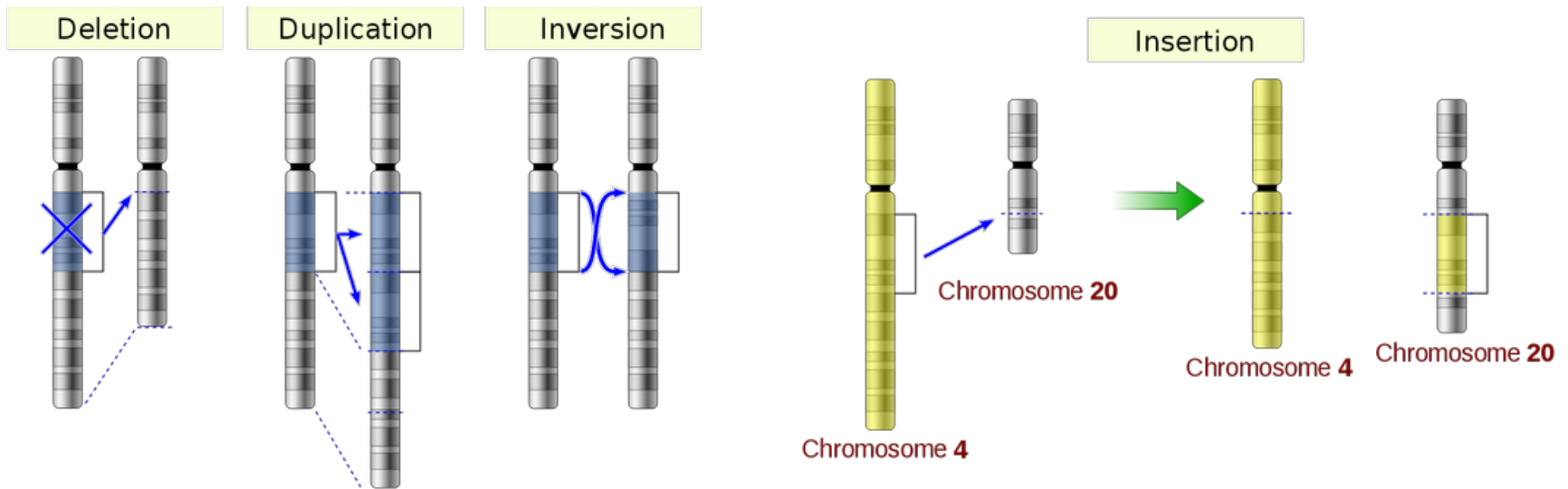
- More than one RNA molecule from one transcript
- Reading directions of the RNA molecules are independent
- Splicing allows deletions of markers



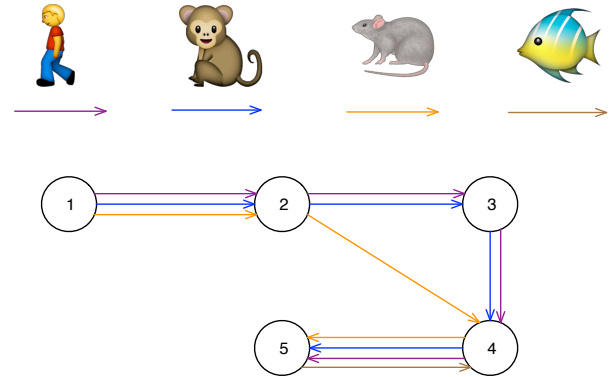
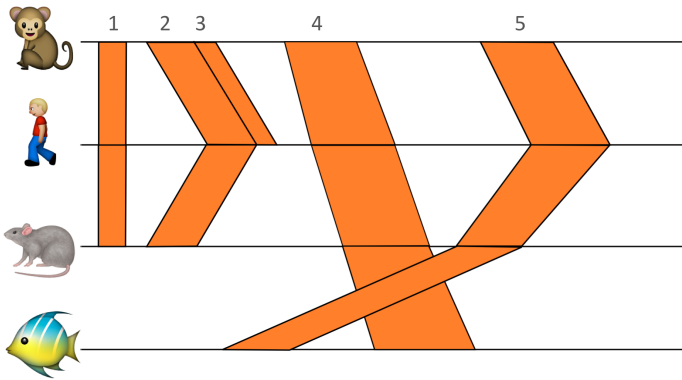
TOTALLY UNRELATED RNA EXAMPLE OF BETWEENNESS



- It is a Betweenness Optimization Problem
 - The direction is not clear
 - Because of circularity, not all triples can be fulfilled
- Find largest subset of Triples that can be fulfilled by a linear order
- The linear order is the most likely linear transcript



- Supergenome Problem is a Betweenness Optimization Problem
- Chromosome mutations
 - Direction of blocks is not clear
 - Not all triples can be fulfilled



- Create graph from alignment
- One block is one vertex
- A edge from block \mathbf{v} to \mathbf{w} in color \underline{x} is added if block \mathbf{w} is successor of \mathbf{v} in a genome \underline{x}



- Extends betweenness problem to a graph.
- Colored Multigraph Betweenness Problem
 - Find a maximal subset of colored edges E' of the multigraph such that the set of triples $C(E')$ has a total order, where $(i,j,k) \in C(E')$ if and only if there are two edges $\{i,j\}$ and $\{j,k\}$ with the same color.

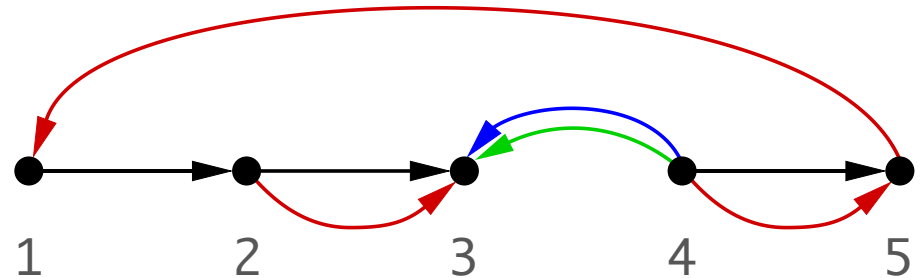
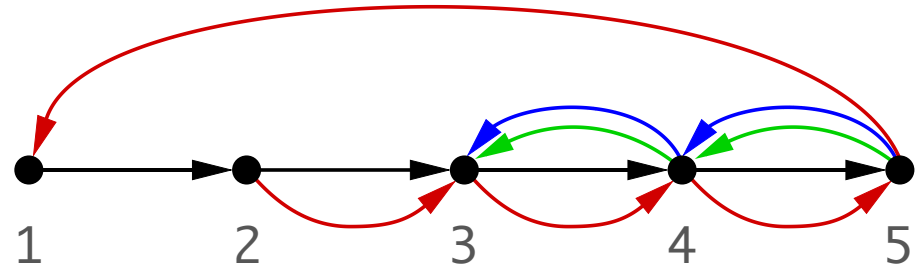
- Idea to solve this is:



- Create a order of the vertices of the graph.
- Then calculate which edges are in the subset.

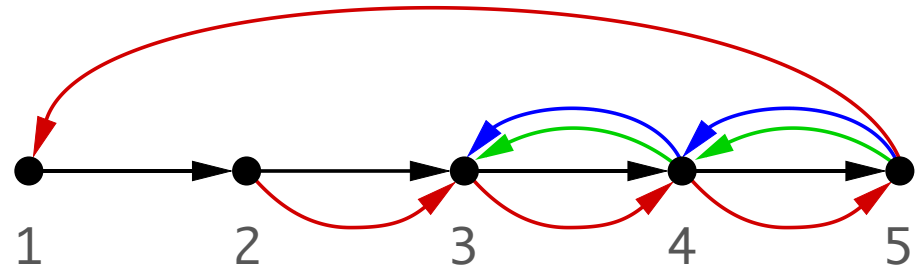
- Topological Sorting
 - Create order out of a DAG
 - If there is an edge from \mathbf{v} to \mathbf{w} , than \mathbf{v} is before \mathbf{w} in the order

- Feedback Arc Sets (FAS)
 - Create a DAG
 - Remove as less edges as possible



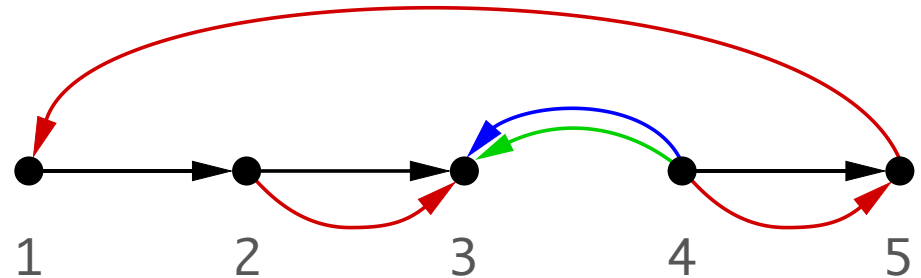
Order: 4, 5, 1, 2, 3

- The resulting order destroys many betweenness information
- FAS does not fit betweenness problem well
 - Create artificial sinks and sources

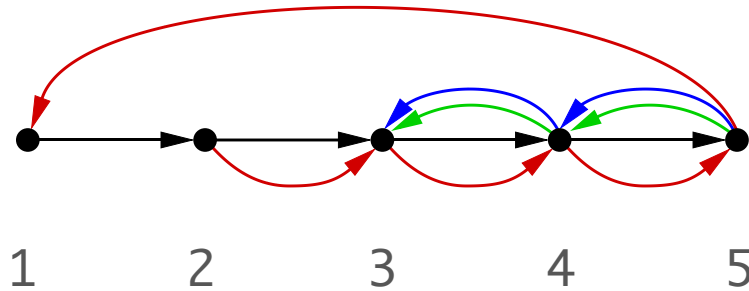


(1, 2, 3)

(4, 5, 1)



Order: 4, 5, 1, 2, 3



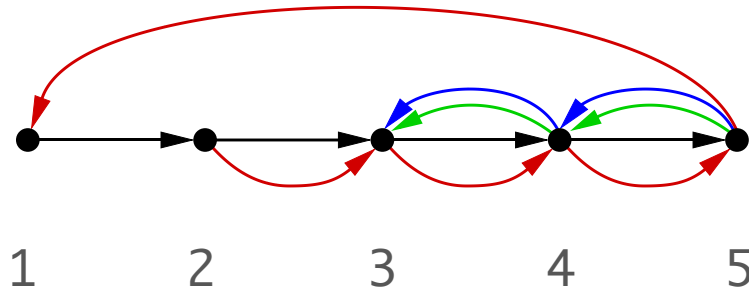
(1,2,3) (2,3,4) (3,4,5)

(2,3,4) (3,4,5) (4,5,1)

(5,4,3)

(5,4,3)





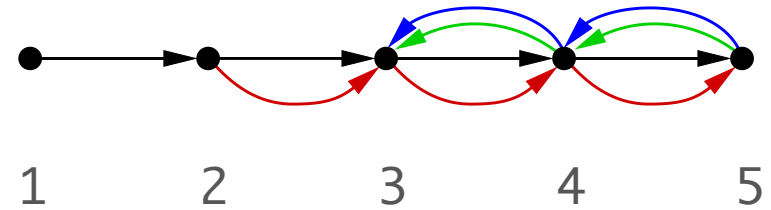
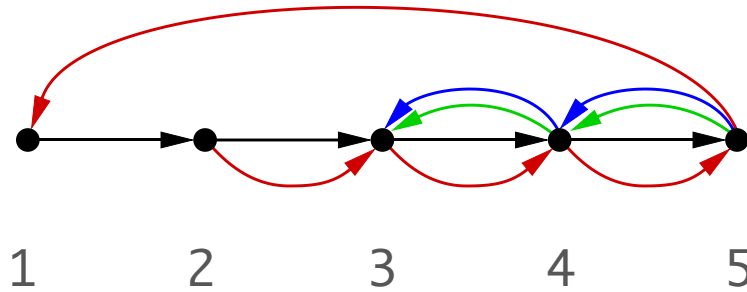
(1,2,3) (2,3,4) (3,4,5)

(2,3,4) (3,4,5) (4,5,1)

(5,4,3)

(5,4,3)





(1,2,3) (2,3,4) (3,4,5)

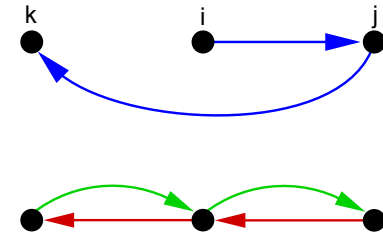
(2,3,4) (3,4,5)

(5,4,3)

(5,4,3)

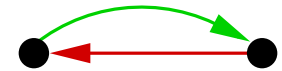


- Betweenness allow some cycles.
- Remove of all cycles is too much
- Two types of cycles:
 - Inconsistent cycles
 - Undirected cycles
- If only inconsistent cycles in the graph
 - Solution to FAS and Betweenness Problem is the same

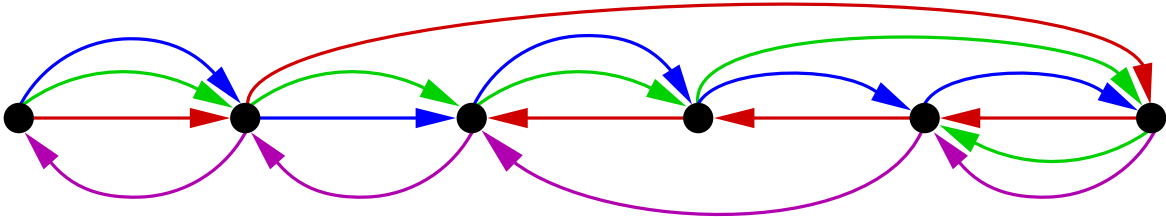


- Remove (most of) the undirected cycles
 - Mini-cycle Remover
- FAS is NP-complete
 - Use heuristic
- Noise reduction by simplifying collinear parts of the graph.
 - Sink/source simplifier
 - Closed-DAG simplifier

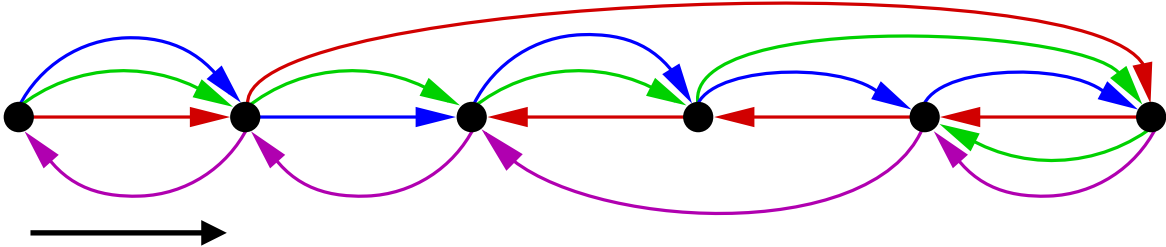
- Type of the cycle depends on used total order
- No order is given at this point
 - Use heuristic to find undirected cycles
- A mini-cycle with only two vertices
 - Very likely a undirected cycles
- Remove all mini-cycles in a intelligent way
 - If two mini-cycles share a vertex remove them together
 - Avoids generation of artificial sinks and sources



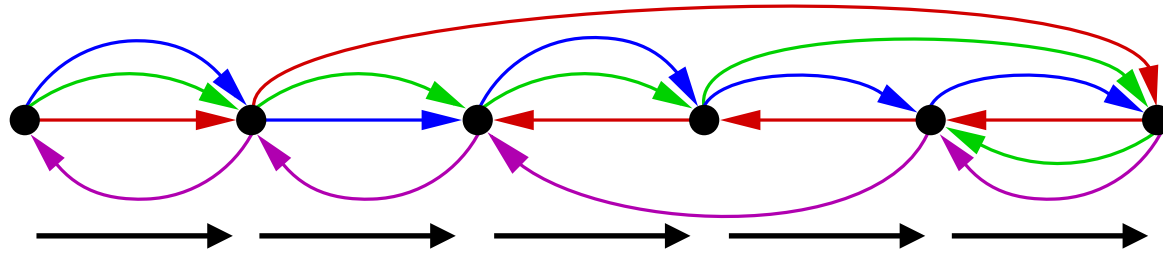
MINI-CYCLE REMOVER EXAMPLE



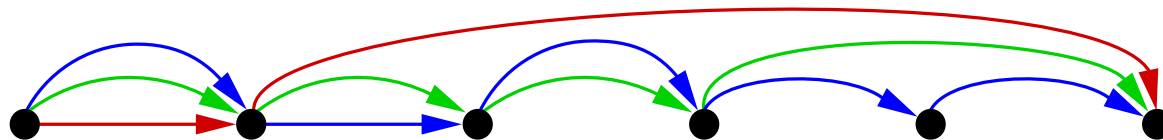
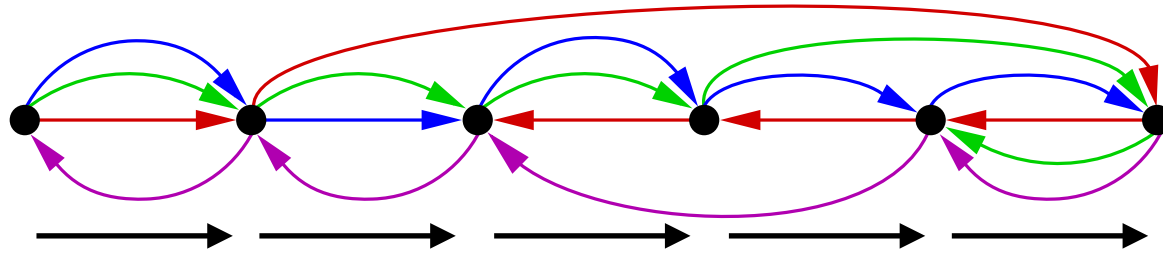
MINI-CYCLE REMOVER EXAMPLE



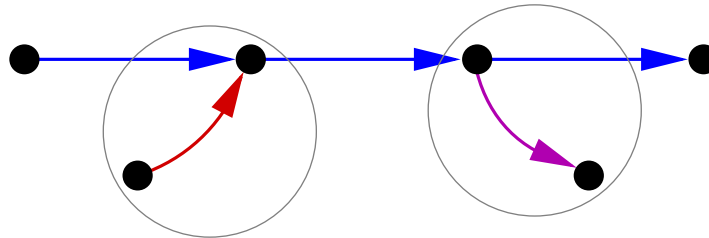
MINI-CYCLE REMOVER EXAMPLE



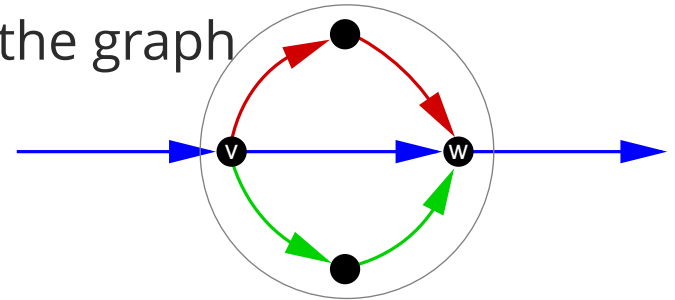
MINI-CYCLE REMOVER EXAMPLE

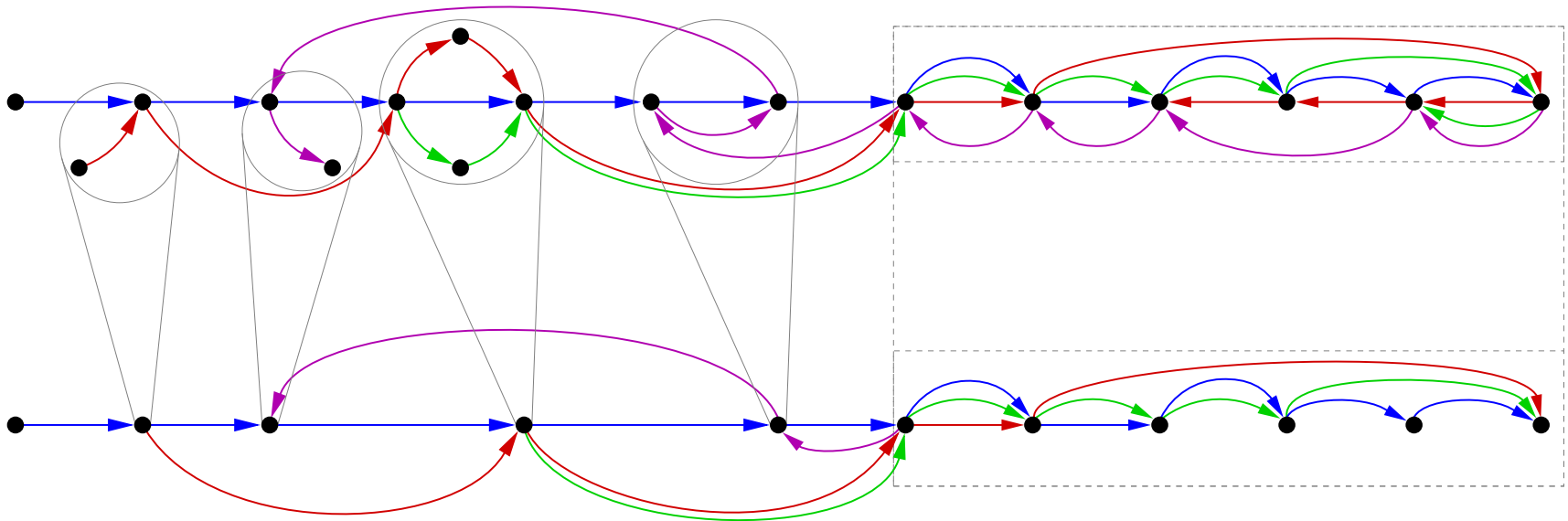


- A sink/source with only one predecessor/successor
- The position in the order is only influenced by the predecessor/successor
- It can be placed directly behind/before this predecessor/successor
- This is a collinear part in the graph
 - Simplified to one vertex



- A Closed-DAG is a collinear part in the graph
- It has this features:
 - It is a directed acyclic graph
 - It is connected to the rest of the graph by a single source vertex **v** and a single sink vertex **w**
 - All direct successors of **v** and all direct predecessors of **w** are contained in it
 - All vertices in it are successors of **v** and predecessors of **w**
- The Closed-DAG is an atomic unit in the order.







- Directed acyclic graph
 - Remove all edges that go from a vertex on position i to a vertex on position j if $j < i$
- Betweenness graph
 - Add inverses Edges
 - Readd all edges that do not create a bad triple (i,j,k)



- Number of edges and triples can be counted
- No gold standard!
 - Can be compared with the start graph





- Two UCSC Datasets.
 - Created with a Reference Species
- Yeast
 - 7 species
 - 43495 vertices
 - 203275 edges, 197043 triples
- Insects
 - 27 species
 - 1451433 vertices
 - 25549792 edges, 25540919 triples





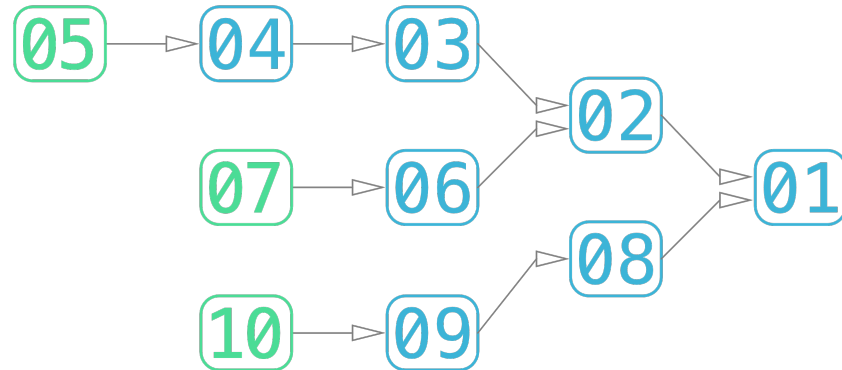
<i>Edges % (triples %)</i>	<i>Yeast DAG</i>	<i>Insect DAG</i>	<i>Yeast Betweenness</i>	<i>Insect Betweenness</i>
<i>Simple FAS</i>				
<i>No mini-cycle Remover</i>				
<i>No simplifier</i>				
<i>All</i>				



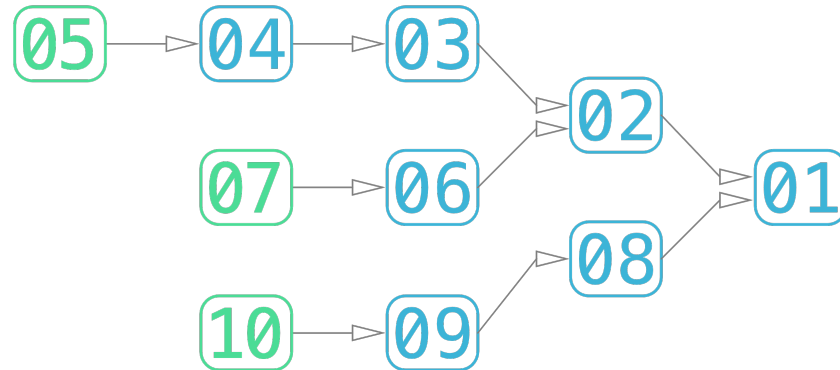
<i>Edges % (triples %)</i>	<i>Yeast DAG</i>	<i>Insect DAG</i>	<i>Yeast Betweenness</i>	<i>Insect Betweenness</i>
<i>Simple FAS</i>	66.87 (53.86)	61.54 (52.49)	82.65 (66.56)	86.97 (75.24)
<i>No mini-cycle Remover</i>	66.86 (53.87)	61.55 (52.50)	82.67 (66.60)	86.98 (75.26)
<i>No simplifier</i>	60.99 (59.29)	56.75 (56.37)	96.96 (94.04)	99.39 (98.80)
<i>All</i>	61.04 (59.31)	56.75 (56.37)	96.91 (93.94)	99.39 (98.80)

- Betweenness is everywhere!
- Solve optimization problem
- New graph based solution
 - Maximal subset of Edges
- Well studied approaches does not fit well
 - Can be fixed by a preprocessing
- Results can be measured
- Results look very promising

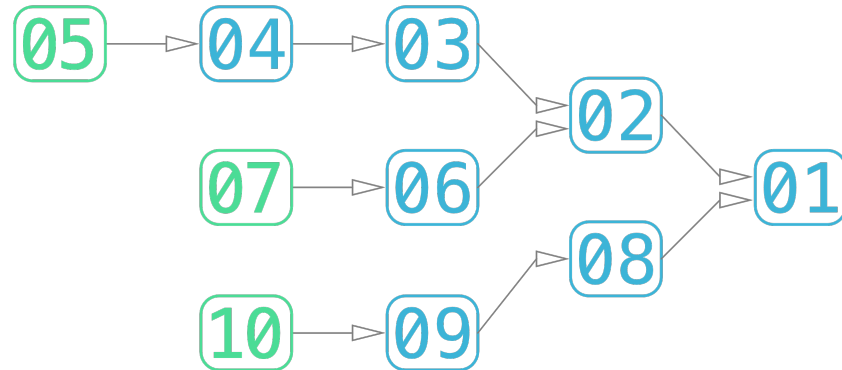
THANK YOU FOR YOUR ATTENTION



- Topological sorting is not unambiguous
- Valid orders e.g.:
 - 5,4,3,7,6,2,10,9,8,1
 - 5,7,10,4,9,6,3,8,2,1
 - 10,7,5,6,4,3,2,9,8,1
 - 7,10,5,4,9,3,6,8,2,1



- Use Distance information
- Next vertex in order is chosen by distance
- Not optimal for betweenness
- Valid orders e.g.:
 - 5,4,3,7,6,2,10,9,8,1
 - 7,6,5,4,3,2,10,9,8,1
 - 10,9,8,7,6,5,4,3,2,1



- Optimize minimal number of violation of the Robinson rule (1951):

$$\max(d(i,j),d(j,k)) \leq d(i,k)$$

$$\underbrace{i < j < k}_{\leq}$$

- Change the order to an other valide topological sorting
- Check if number of violation is lowered
- Reaped until no further optimization is found
- 7,6,5,4,3,2,10,9,8,1** \longrightarrow **5,4,3,7,6,2,10,9,8,1**



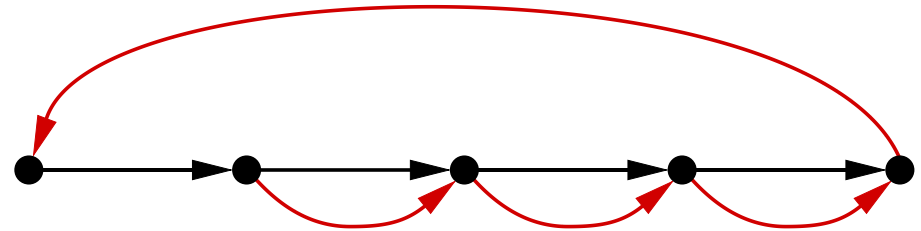
- 1218 plagiarism fragments
- 135 sources
- 63% of the work
- Sources widely distributed
- Possible questions:
 - Is basic structure from a source?
 - Which source is dominant in which part?





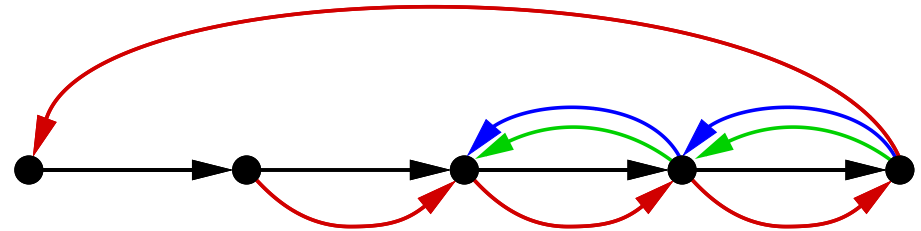
- 1218 plagiarism fragments
- 135 sources
- 63% of the work
- Sources widely distributed
- Possible questions:
 - Is basic structure from a source?
 - Which source is dominant in which part?





- Cites, pages, or sections as vertices
- Edges in the order of the dissertation and in order of the cites.





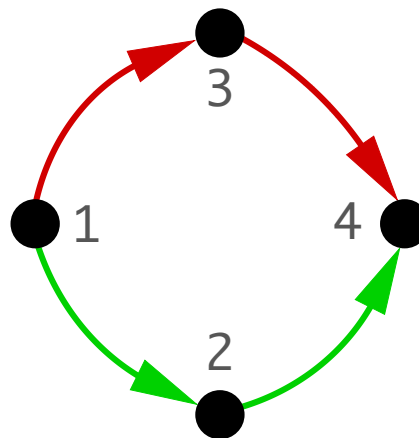
- Cites, pages, or sections as vertices
- Edges in the order of the dissertation and in order of the cites.



- A Hamiltonian path is a path that visits each vertex exactly once.
- The graph is connected
- Ignoring the direction of edges
 - Betweenness has no direction

1, 2, 4, 3

1, 3, 4, 2

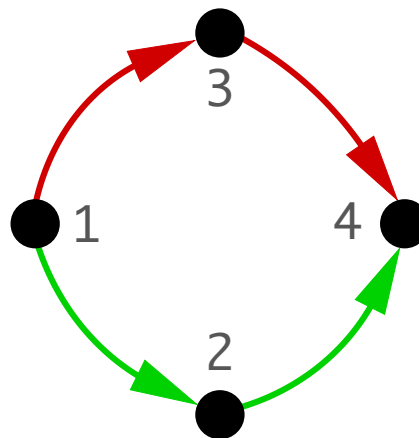




- Violated betweenness when two parts parallel
- Does not fit betweenness problem well

1,2,4,3

1,3,4,2



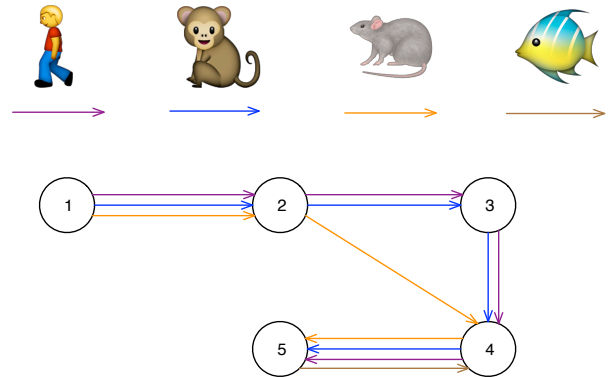
Betweenness solution:

1,2,3,4

1,3,2,4



	1	2	3	4	5
1-2	1	1	0	0	0
2-3	0	1	1	0	0
2-4	0	1	0	1	0
3-4	0	0	1	1	0
4-5	0	0	0	1	1
5-4	0	0	0	1	1



- Matrix with vertices as columns and adjacencies as rows
- Sort both the rows and columns of the matrix independently
- In such a way that rows and columns show all non-zero entries consecutively

	1	2	3	4	5
1-2	1	1	0	0	0
2-3	0	1	1	0	0
2-4	0	1	0	1	0
3-4	0	0	1	1	0
4-5	0	0	0	1	1
5-4	0	0	0	1	1

	1	2	3	5	4
3-4	0	0	1	0	1
2-3	0	1	1	0	0
1-2	1	1	0	0	0
2-4	0	1	0	0	1
4-5	0	0	0	1	1
5-4	0	0	0	1	1

	1	3	2	4	5
1-2	1	0	1	0	0
2-3	0	1	1	0	0
3-4	0	1	0	1	0
2-4	0	0	1	1	0
4-5	0	0	0	1	1
5-4	0	0	0	1	1

- Consecutive ones property is violated even when betweenness is intact
- Bad adjacencies have huge impact
- Does not fit betweenness problem well