Reduction of Graph-Grammar Models by grouping of tautomers

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Overview

In this presentation, we will be talking about:

- Sugar chemistry, in particular, the formose chemistry
- Equivalence Classes
- Auto catalysis
- Gillespie's exact stochastic simulation algorithm
- Bisimulation
- Markov Chains

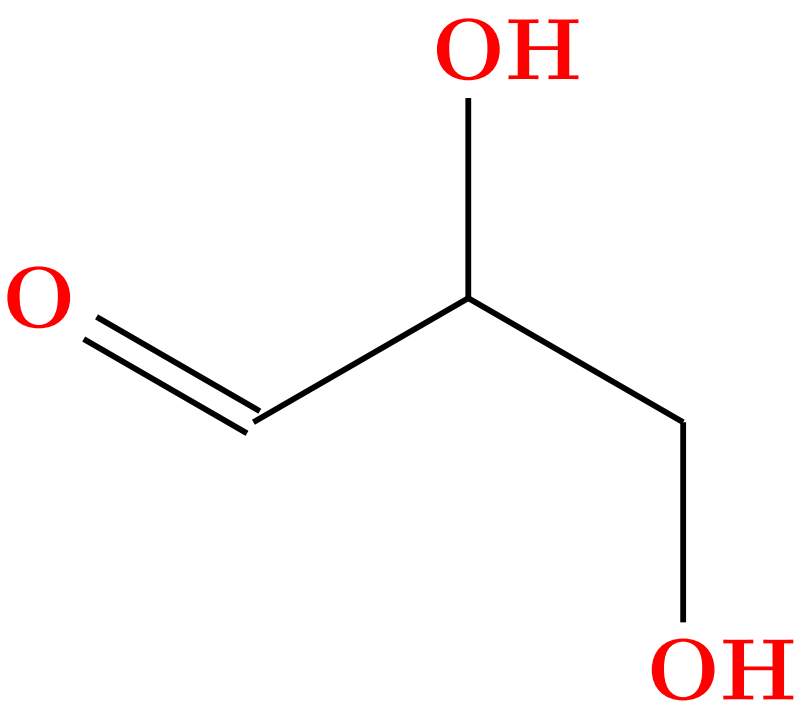
What are we doing?

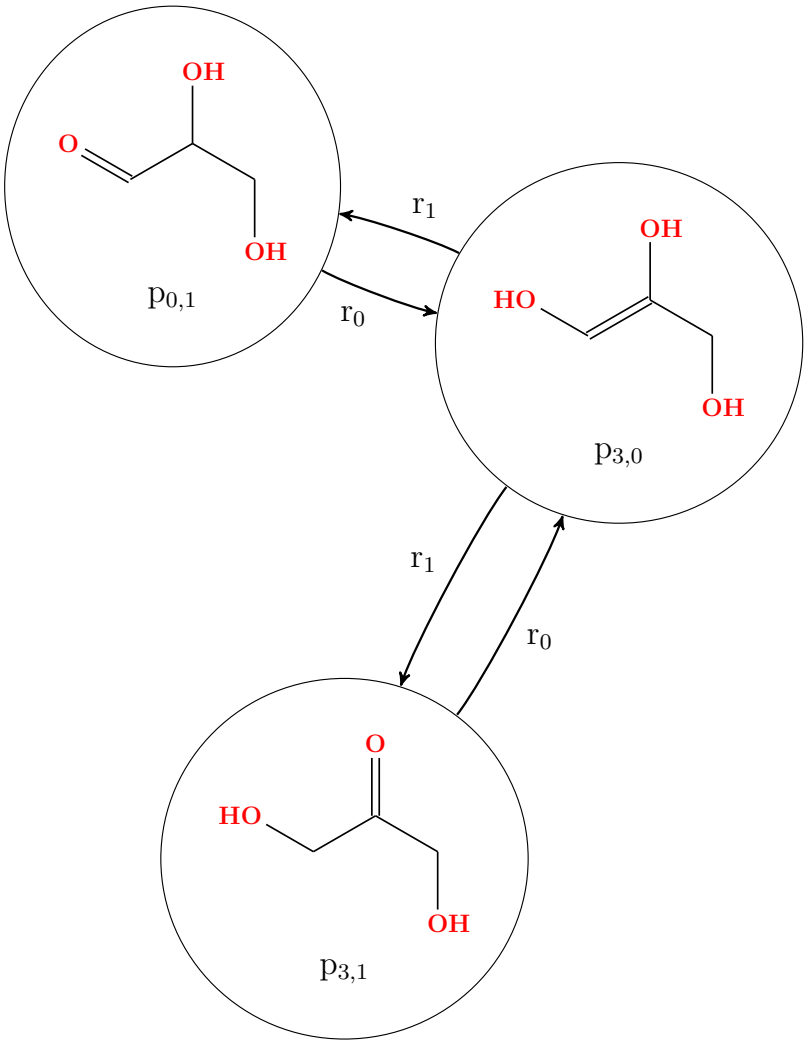
- Derivation graphs can get LARGE
- Many nodes may have extremely similar properties
- Stack these nodes together in a projection

Why do this?

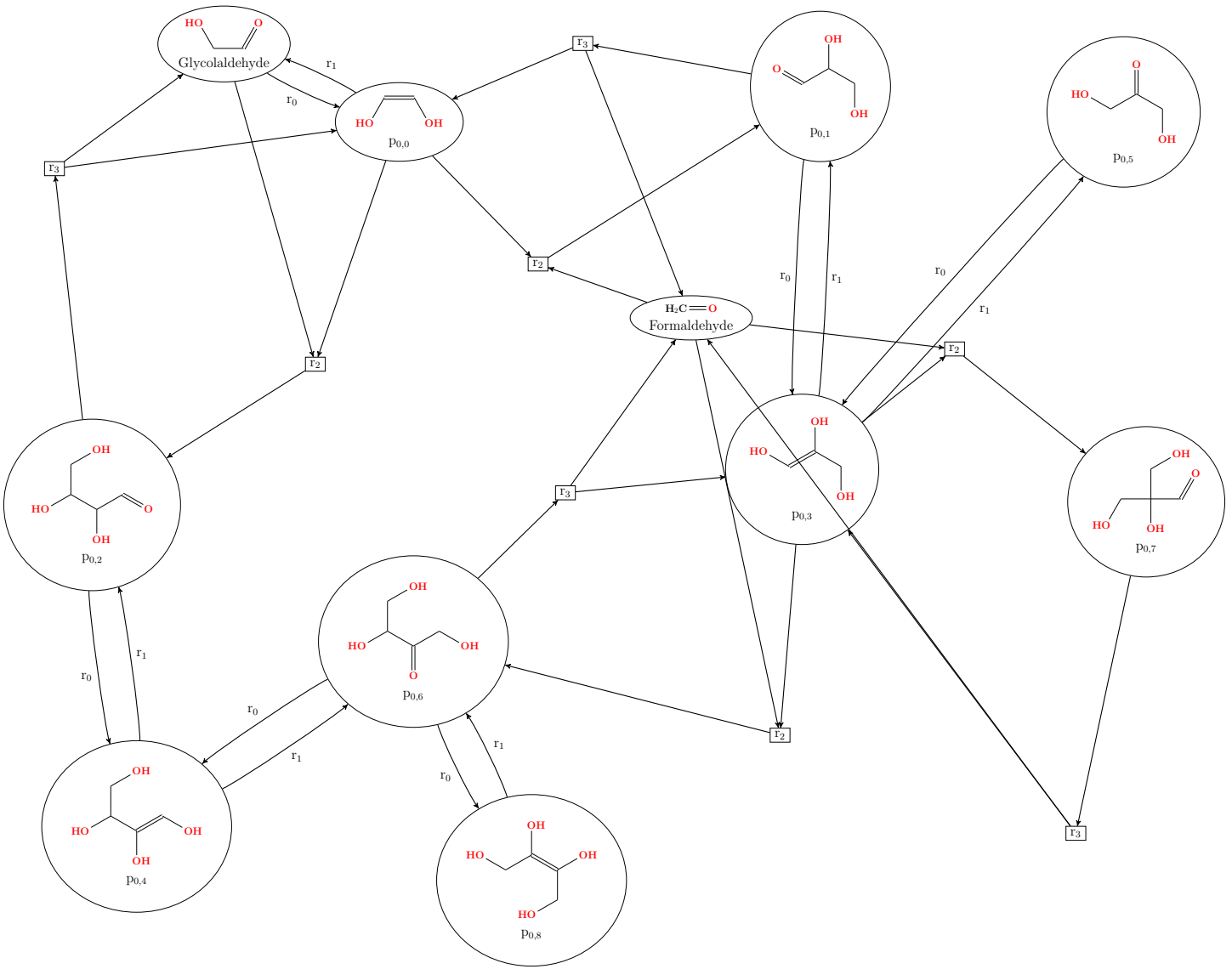
- Projected derivation graphs have much fewer nodes
- Much faster auto catalysis computation, and more correct results
- Stochastic simulations run significantly faster, and potentially produce more correct results

Equivalence Class in practice

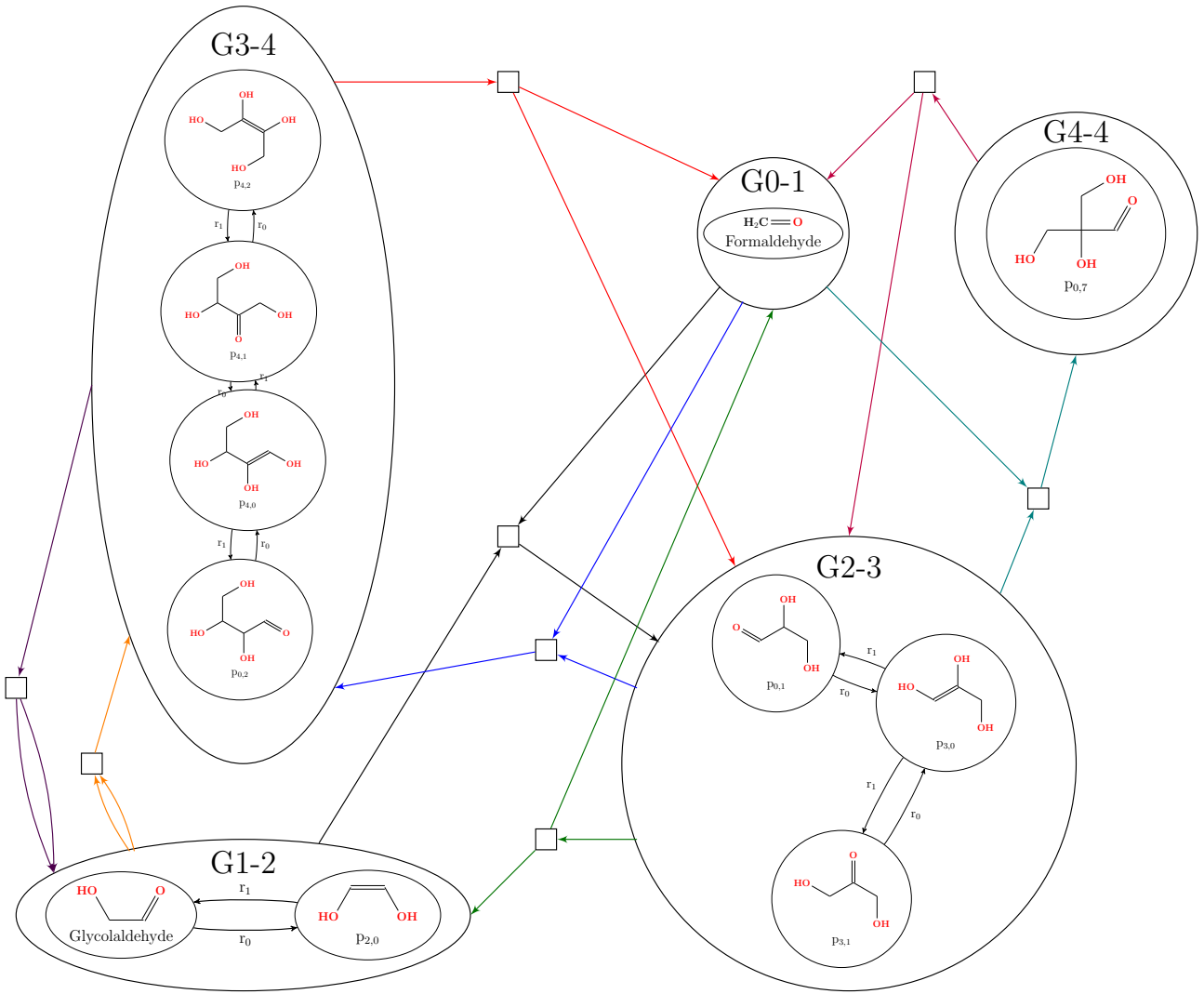




Formose reaction autocatalysis example: Unprojected



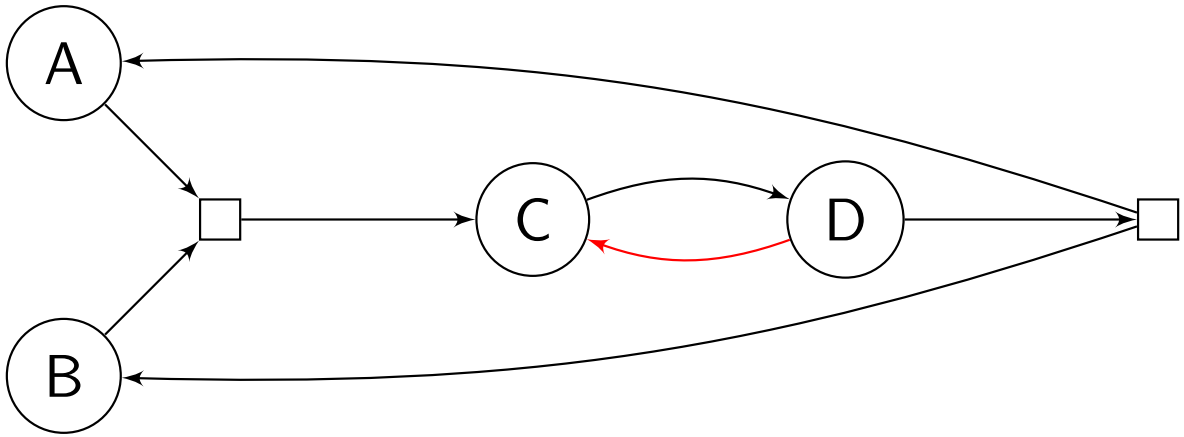
Formose reaction autocatalysis example: Projection

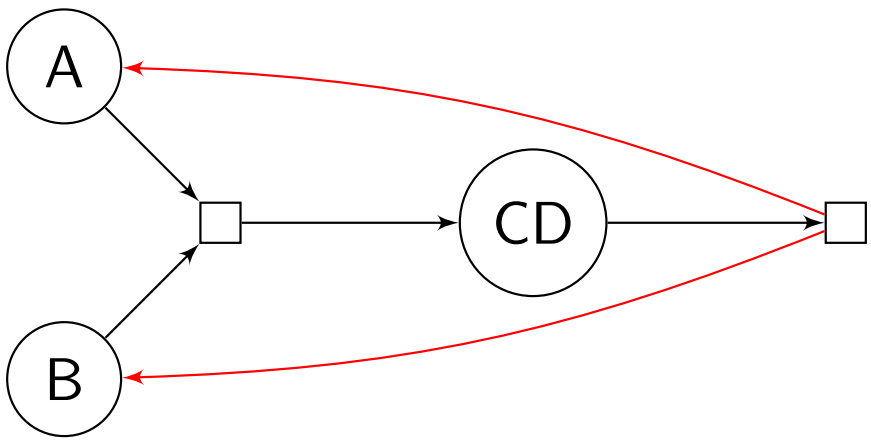


Equivalence Class Theory

- $\{x \in X : xRy\}$
- y is an element of X
- The notation "xRy" means there is an equivalence relation between x and y
- for all $x, y \in X$, we have $xRy \Leftrightarrow x$ and y belong to the same equivalence class

Why is projected auto catalysis more correct?





Why could projected stochastic simulation be more correct?

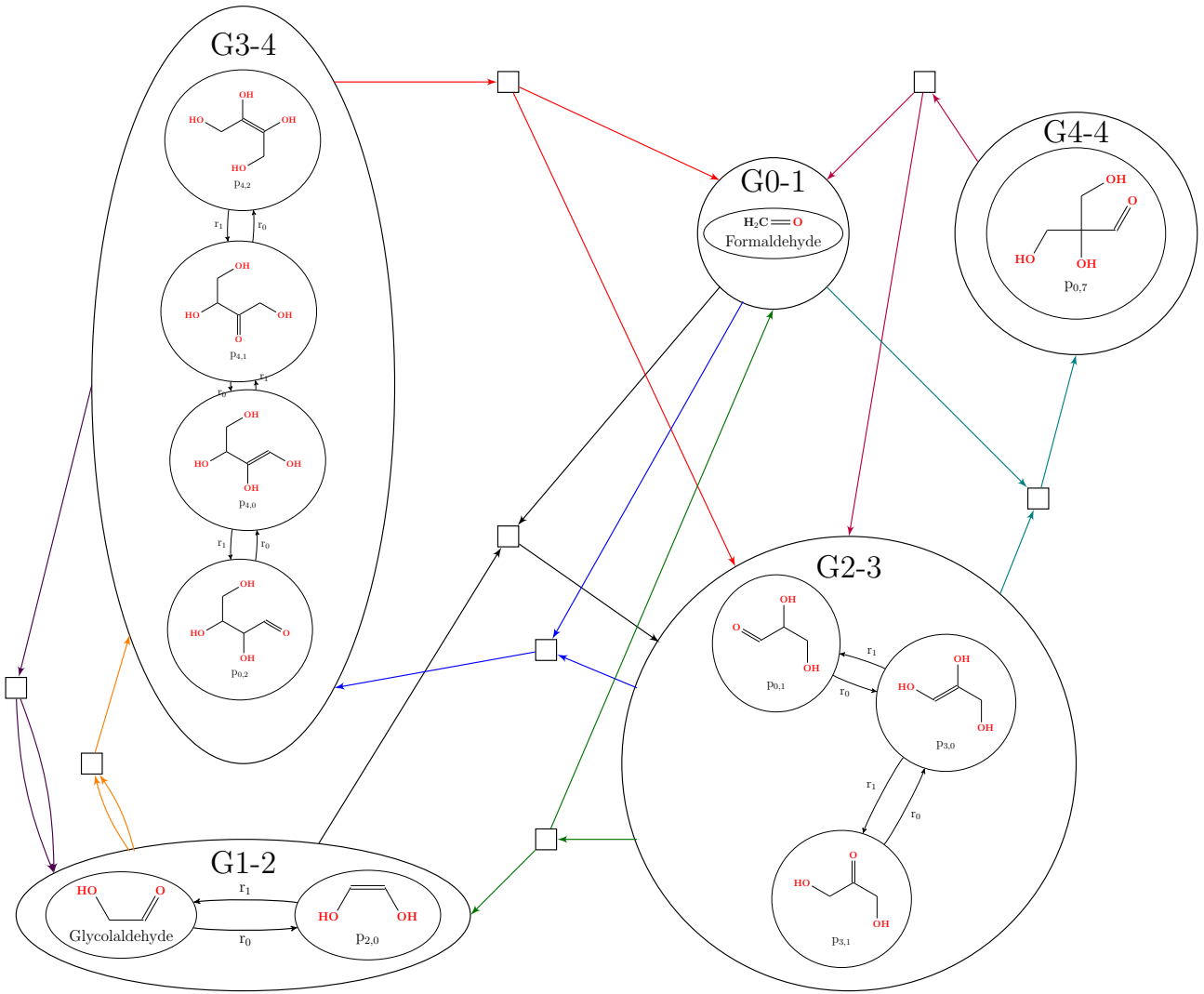
- Keto-Enol reactions are said to be extremely fast, especially comapred to Aldol-addition reactions
- Getting a realistic relative reaction rate, and also a reasonable computation time, is not possible normally
- "Removing" the Keto-Enol reactions from the equation makes it much easier to get something closer to real life

Concurrency Theory: Bisimulations

Bisimilarity definitions:

- *Strong Bisimilarity*: A binary relation $\mathcal{R}$ over the set of states of an Labelled Transition System is a strong bisimulation if and only if whenever $s_1 \mathcal{R} s_2$ and α is an action:
  - if $s_1 \xrightarrow{\alpha} s_1'$, then there is a transition $s_2 \xrightarrow{\alpha} s_2'$ such that $s_1' \mathcal{R} s_2'$, and
  - if $s_2 \xrightarrow{\alpha} s_2'$, then there is a transition $s_1 \xrightarrow{\alpha} s_1'$ such that $s_1' \mathcal{R} s_2'$
- *Weak Bisimilarity*: A binary relation $\mathcal{R}$ over the set of states of a Labelled Transition System is a weak bisimulation if and only if whenever $s_1 \mathcal{R} s_2$ and α is an action (Including τ, the internal action):
  - If $s_1 \xrightarrow{\alpha} s_1'$, then there is a transition $s_2 \stackrel{\alpha}{\Rightarrow} s_2'$ such that $s_1' \mathcal{R} s_2'$ and
  - If $s_2 \xrightarrow{\alpha} s_2'$, then there is a transition $s_1 \stackrel{\alpha}{\Rightarrow} s_1'$ such that $s_1' \mathcal{R} s_2'$.

Concurrency Theory: Bisimulations, example



Results

- A "Unique reaction" in the following table means specifically a 1-2 or 2-1 reaction.
- The autocatalysis results are best described by how many different solutions are found and how fast. This is reflected in how much of the table is actually filled out.
- The stochastic simulation results are best described by how fast a simulation reaches its end, and how far the simulation got towards its end.

Autocatalysis results: Unprojected

Cells: How many different solutions
Columns: How many carbons are contained in the largest molecule in the solution

	4 C	5 C	6 C	7 C	8 C	9 C	Sum
3 Unique Reactions	1	0	0	0	0	0	1
4 Unique Reactions	0	2	8	4	6	12	32
5 Unique Reactions	0	32	119	394	927	—	1472
6 Unique Reactions	1	36	412	1640	4880	—	6969
7 Unique Reactions	0	92	2556	25586	—	—	28234
8 Unique Reactions	0	144	10053	137364	—	—	147561
9 Unique Reactions	0	185	45469	—	—	—	45654
10 Unique Reactions	0	239	170536	—	—	—	170775

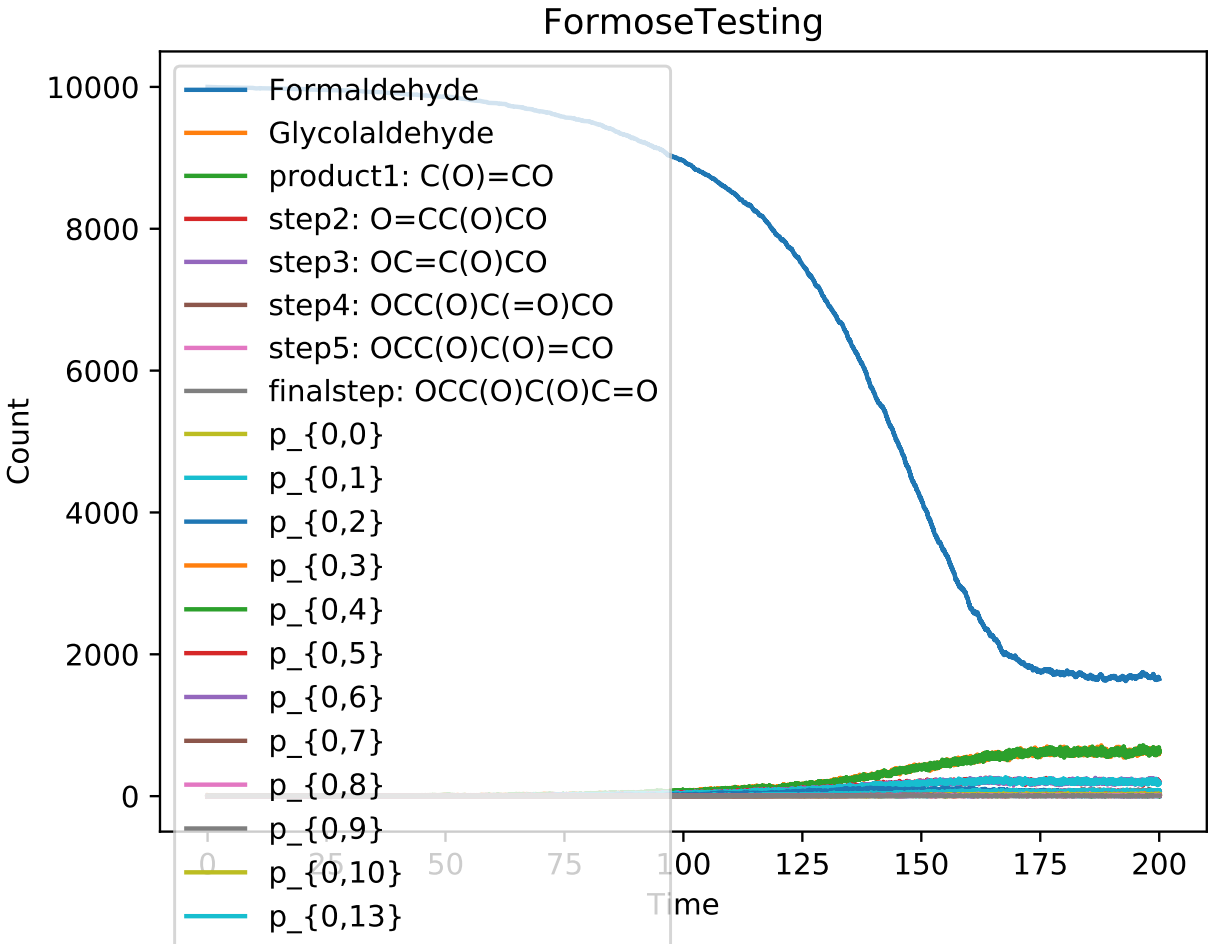
Autocatalysis results: Projection

	4 C	5 C	6 C	7 C	8 C	9 C	Sum
3 Unique Reactions	1	0	0	0	0	0	1
4 Unique Reactions	0	0	2	0	0	0	2
5 Unique Reactions	0	18	29	106	196	357	706
6 Unique Reactions	1	21	162	561	1278	—	2023
7 Unique Reactions	0	57	891	7271	27768	—	35987
8 Unique Reactions	0	102	4012	45817	—	—	49931
9 Unique Reactions	0	125	17529	377398	—	—	395052
10 Unique Reactions	0	166	67407	—	—	—	67573

Stochastic simulation results: Unprojected

Runtime: 81 minutes and 51.395 seconds

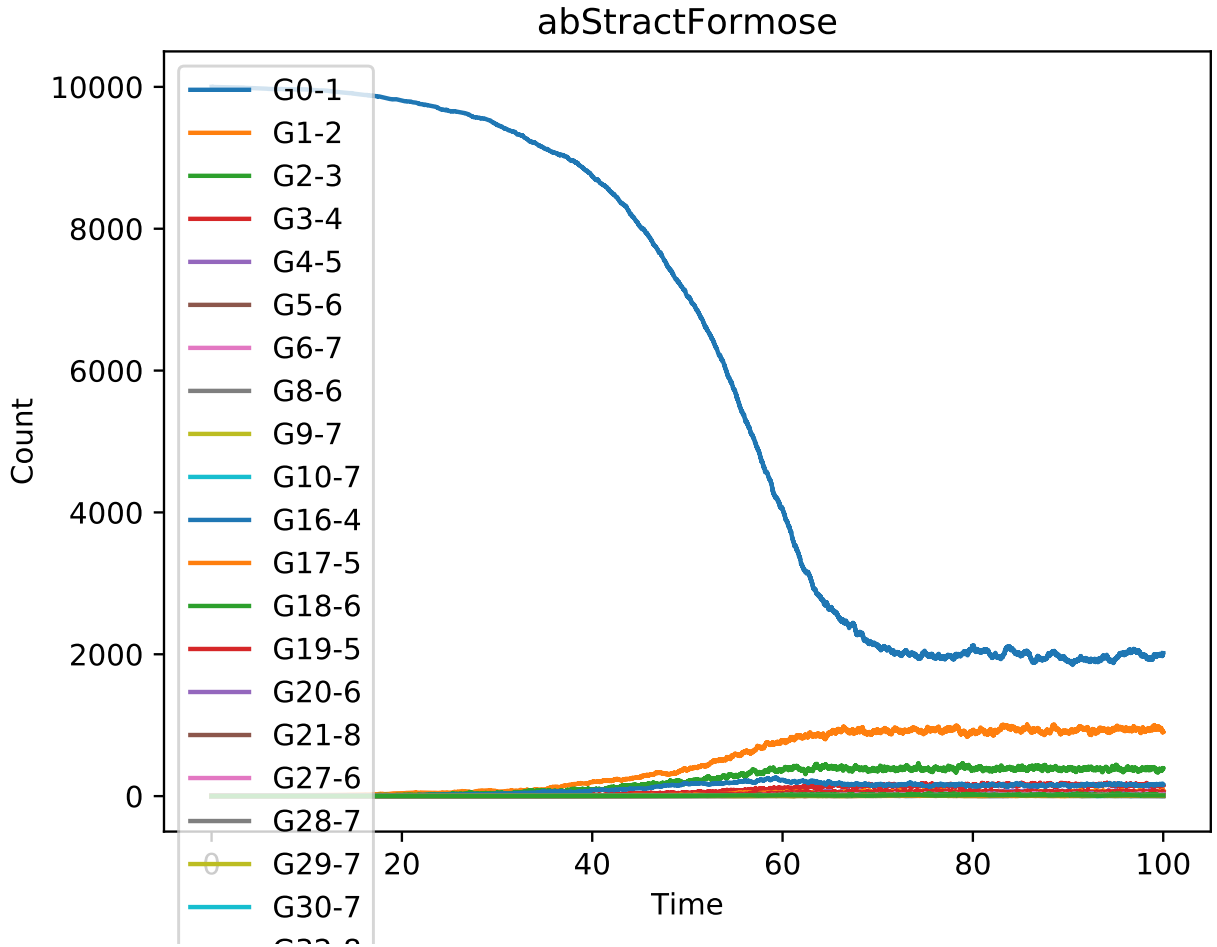
Details: 100 times higher reaction rate on keto-enol than on aldol-addition



Stochastic simulation results: Projected

Runtime: 6 minutes and 26.545 seconds

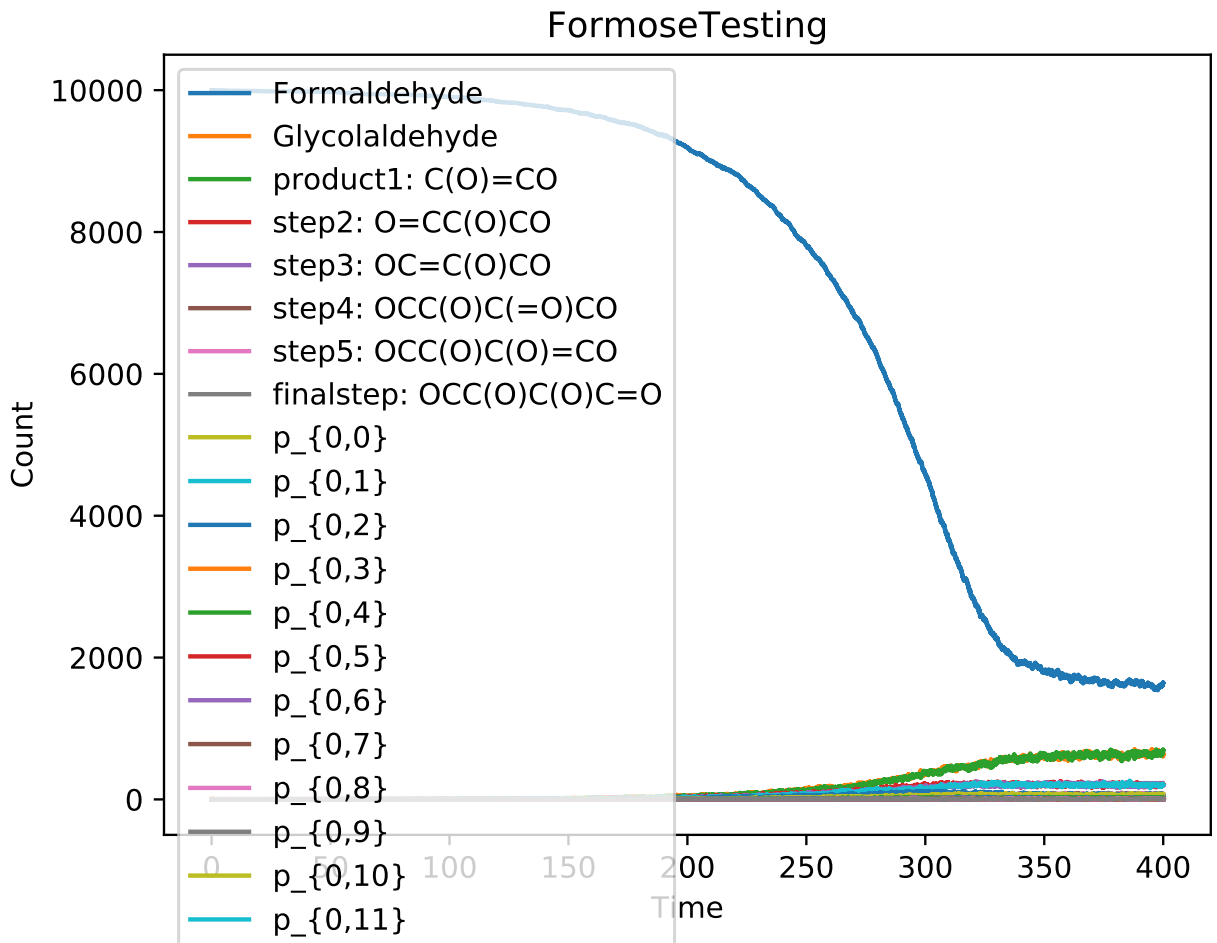
Details: Same reaction rate on aldol-addition as for unprojected



Stochastic simulation results: Unprojected 2

Runtime: 25 minutes and 18.185 seconds

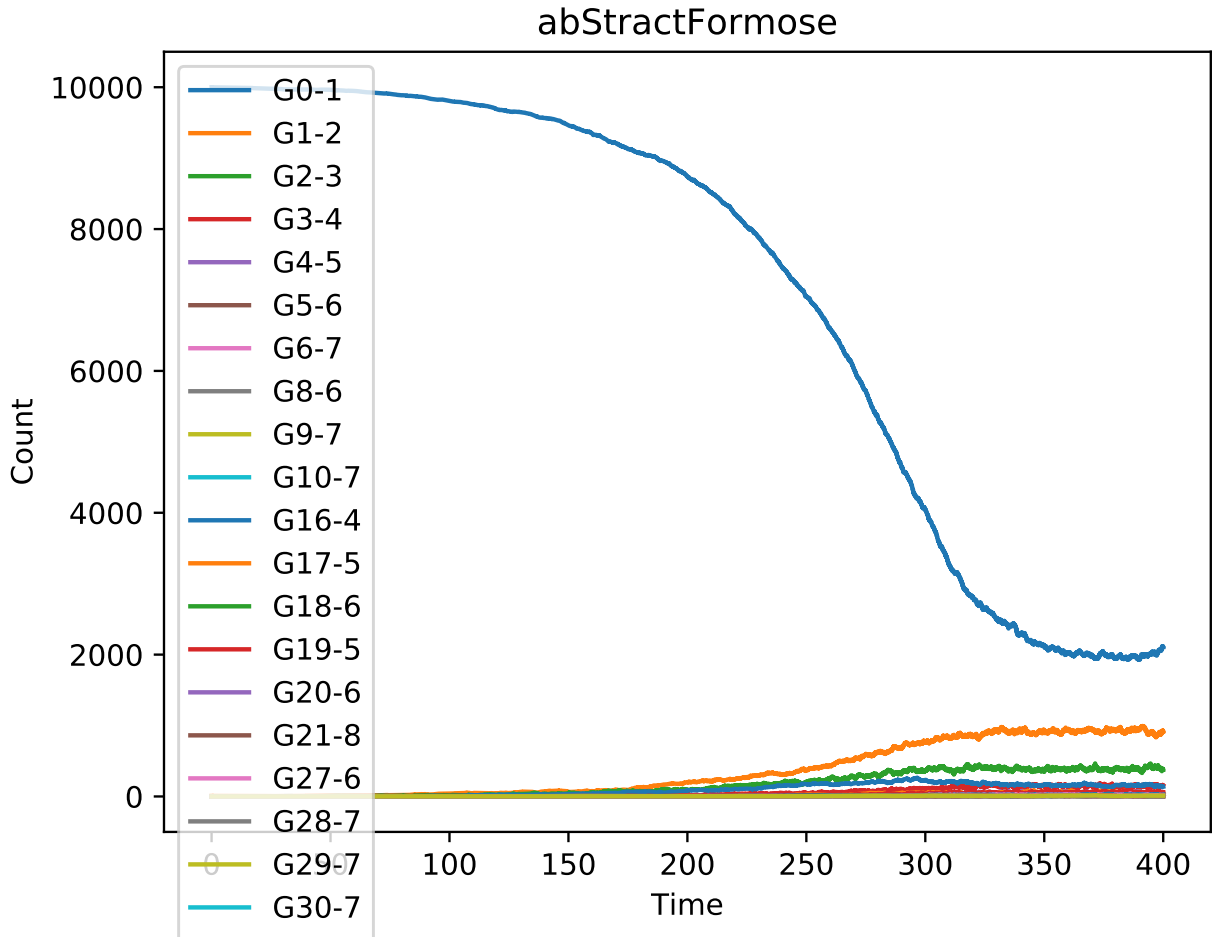
Details: Keto-enol reaction rate same as aldol-addition



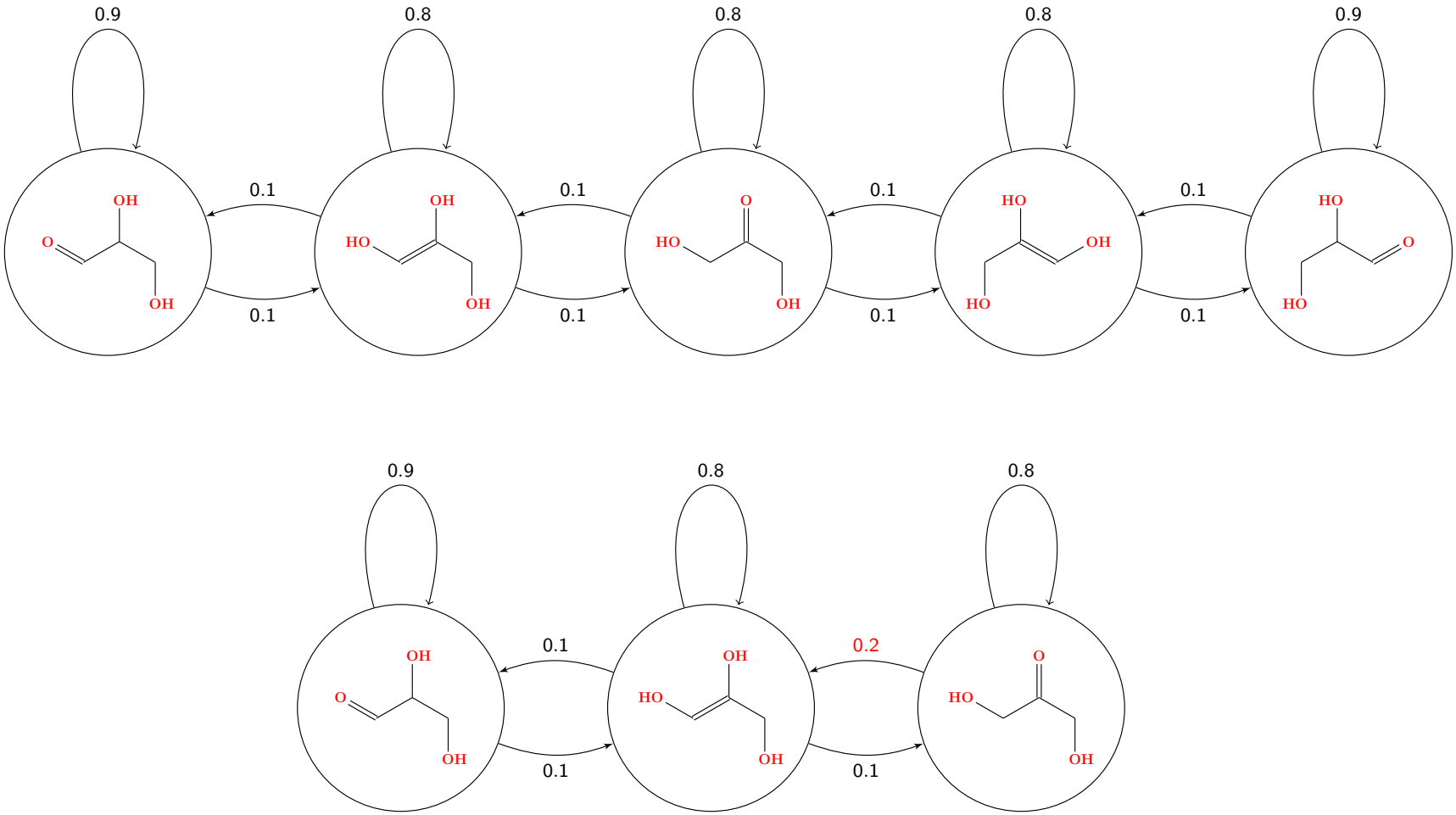
Stochastic simulation results: Projected 2

Runtime: 3 minutes and 42.391 seconds

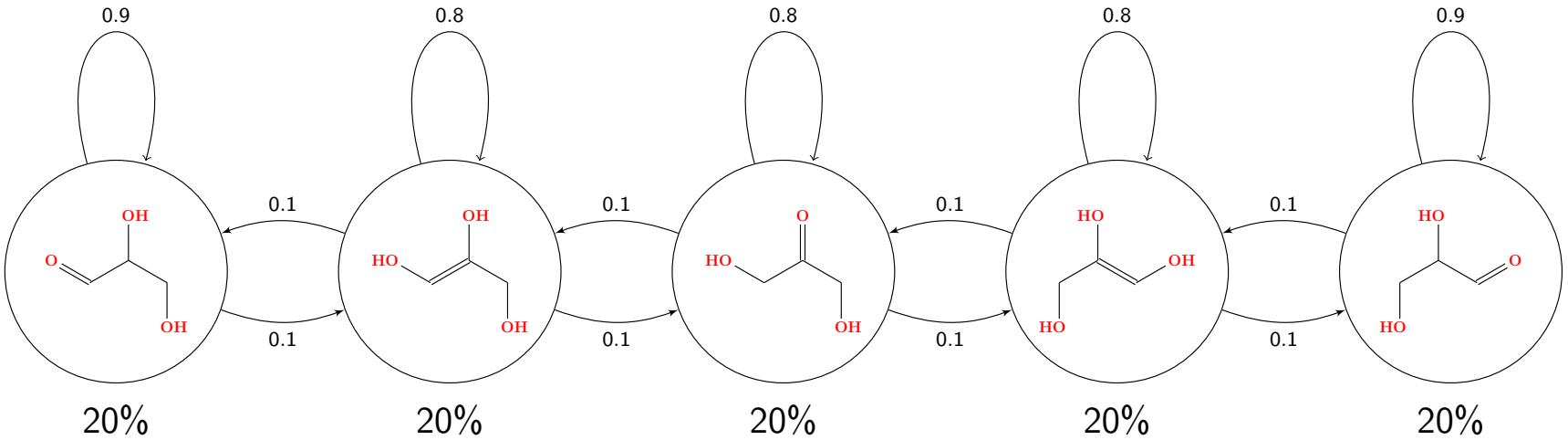
Details: reaction rate 1/5th of unprojected.

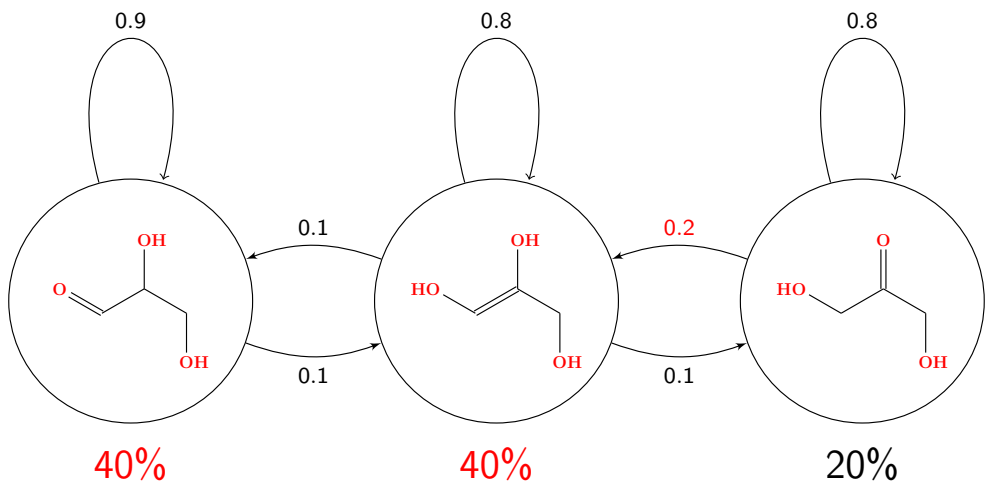


Markov Chains: Example 1: Formose reaction



Markov Chains: Example 1: Formose reaction





Summary

- Equivalence classes can be used to reduce derivation graphs based on (for instance) tautomerisms
- Such projections improve both the computation time and quality of auto catalysis and
- also improves the computation time of Stochastic simulations, and may sometimes also improve the quality of the stochastic simulation itself
- Markov Chains can be used to show that the projection does not appear to cause any negative effect on the concentration differences in such a stochastic simulation
- The comparison between the unprojected and projected solutions can be proven to be weakly bisimilar